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ASYMPTOTIC BEHAVIOR OF SOLUTIONS FOR INCLUSIONS WITH CAUSAL MULTIOPERATORS AND THE METHOD OF INTEGRAL GUIDING POTENTIALS

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Abstract. In the present paper the method of integral guiding potentials is applied to study the problem of the asymptotic behavior of solutions for a differential inclusion with a causal multioperator. At first we consider the case when the multioperator is closed and convex-valued. Then the case of a non-convex-valued and lower semicontinuous right-hand part is considered.
Keywords: functional inclusion; causal multioperator; asymptotic behavior of solutions; integral guiding potential

Introduction

The study of systems governed by differential and functional equations with causal operators, which is due to Tonelli [1] and Tychonov [2], attracts the attention of many researchers. The term causal arises from the engineering and the notion of a causal operator turns out to be a powerful tool for unifying problems in ordinary differential equations, integro-differential equations, functional differential equations with finite or infinite delay, Volterra integral equations, neutral functional equations et al. (see the monograph [3]). In the present paper we apply the method of integral guiding potentials to the investigation of the asymptotic behavior of solutions for a differential inclusion with the multivalued causal operator.

The main ideas of the method of guiding functions were formulated by Krasnosel'skii and Perov in the fifties (see [4, 5]). Being geometrically clear, this method was originally applied to the study of periodic and bounded solutions of ordinary differential equations (see, e.g., [6-8]). Thereafter the method was extended to differential inclusions (see, e.g., [9, 10]), differential inclusion with the causal operator (see, e.g., [11, 12]) and other objects. The sphere of applications was extended to the study of qualitative behavior and bifurcations

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of solutions (see, e.g., [13-16]) and asymptotics of solutions (see, e.g., [17-19]). These and other aspects of the method of guiding functions and its applications, as well as the additional bibliography, may be found in the recent monograph [20].

1. Main concepts

Let (X, d_X) and (Y, d_Y) be metric spaces. By the symbols $P(Y)$ and $K(Y)$ we denote the collections of all nonempty and, respectively, nonempty and compact subsets of the space Y. If Y is a normed space, $Cv(K)$ and $Kv(Y)$ denote the collections of all nonempty convex closed (and, respectively, compact) subsets of Y.

D e f i n i t i o n 1. A multimap $F : X \to P(Y)$ is called upper semicontinuous (u.s.c.) at the point $x \in X$ if for each open set $V \subset Y$ such that $F(x) \subset V$ there exists $\delta > 0$ such that $d_X(x, x') < \delta$ implies $F(x') \subset V$. A multimap $F : X \to P(Y)$ is called u.s.c. if it is u.s.c. at each point $x \in X$.

D e f i n i t i o n 2. A multimap $F : X \to P(Y)$ is called lower semicontinuous (l.s.c.) at a point $x \in X$, if for each open set $V \subset Y$ such that $F(x) \cap V \neq \emptyset$ there exists $\delta > 0$ such that $d_X(x, x') < \delta$ implies $F(x') \cap V \neq \emptyset$. A multimap $F : X \to P(Y)$ is called l.s.c. if it is l.s.c. at each point $x \in X$.

D e f i n i t i o n 3. Let I be a closed subset of $\mathbb{R}$ endowed with the Lebesgue measure. A multifunction $F : I \to K(Y)$ is called measurable if, for each open subset $W \subset Y$, its pre-image $F^{-1}(W) = \{t \in I : F(t) \subset W\}$ is the measurable subset of I.

R e m a r k 1. A u.s.c. multifunction is measurable. Each measurable multifunction $F : I \to K(Y)$ has a measurable selection, i.e., there exists such measurable function $f : I \to Y$, that $f(t) \in F(t)$ for a.e. $t \in I$.

Let $T > 0$ and $\sigma \geq 0$ be given numbers. By the symbols $C([kT - \sigma, (k+1)T]; \mathbb{R}^n)$ and $L^1((kT, (k+1)T); \mathbb{R}^n)$, where $k \in \mathbb{Z}$, we will denote the corresponding spaces of continuous and integrable functions with usual norms.

For any subset $\mathcal{N} \subset L^1((kT, (k+1)T); \mathbb{R}^n)$ and $\tau \in (kT, (k+1)T)$ we define the restriction of $\mathcal{N}$ on (kT, τ) as

$$\mathcal{N} |_{(kT,\tau)} = \{f |_{(kT,\tau)} : f \in \mathcal{N}\}.$$

D e f i n i t i o n 4. We will say that $\mathcal{Q}$ is a causal multioperator if for each $k \in \mathbb{Z}$ a multimap

$$\mathcal{Q} : C([kT - \sigma, (k+1)T]; \mathbb{R}^n) \multimap L^1((kT, (k+1)T); \mathbb{R}^n)$$

is defined in such a way that for each $\tau \in (kT, (k+1)T)$ and for all

$$u(\cdot), v(\cdot) \in C([kT - \sigma, (k+1)T]; \mathbb{R}^n)$$

the condition $u |_{[kT-\sigma,\tau]} = v |_{[kT-\sigma,\tau]}$ implies $\mathcal{Q}(u) |_{(kT,\tau)} = \mathcal{Q}(v) |_{(kT,\tau)}$.

Denote by $\mathcal{C}$ the Banach space $C([-\sigma, 0]; \mathbb{R}^n)$.

E x a m p l e 1. Suppose that a multimap $F : \mathbb{R} \times \mathcal{C} \to Kv(\mathbb{R}^n)$ satisfies the following conditions:

$(F1)$ the multifunction $F(\cdot, c) : \mathbb{R} \to Kv(\mathbb{R}^n)$ admits a measurable selection for every $c \in \mathcal{C}$;

$(F2)$ the multimap $F(t, \cdot) : \mathcal{C} \to Kv(\mathbb{R}^n)$ is u.s.c. for a.e. $t \in \mathbb{R}$;

$(F3)$ for every $r > 0$ there exists a locally integrable non-negative function $\eta_r(\cdot) \in L^1_{loc}(\mathbb{R})$ such that

$$\|F(t, c)\| := \sup\{\|y\| : y \in F(t, c)\} \le \eta_r(t) \quad \text{a.e. } t \in \mathbb{R},$$

for all $c \in \mathcal{C}$, $\|c\| \le r$.

It is known (see, e.g., [9, 21]) that under conditions $(F1)$ – $(F3)$ for each $k \in \mathbb{Z}$, the superposition multioperator $\mathcal{P}_F : C([kT - \sigma, (k+1)T]; \mathbb{R}^n) \multimap L^1((kT, (k+1)T); \mathbb{R}^n)$,

$$\mathcal{P}_F(u) = \left\{ f \in L^1((kT, (k+1)T]; \mathbb{R}^n) : \ f(t) \in F(t, u_t) \quad \text{a.e. } t \in (kT, (k+1)T) \right\}$$

is well defined. Here $u_t \in \mathcal{C}$ is defined as $u_t(\theta) = u(t + \theta)$, $\theta \in [-\sigma, 0]$. It is easy to see that the multioperator $\mathcal{P}_F$ is causal.

E x a m p l e 2. Let $F : \mathbb{R} \times \mathcal{C} \to Kv(\mathbb{R}^n)$ be a multimap satisfying conditions $(F1)$ – $(F3)$ of Example 1. Suppose that $\{K(t, s) : -\infty < s \le t < +\infty\}$ is a continuous (with respect to the norm) family of linear operators in $\mathbb{R}^n$ and $m \in L^1_{loc}(\mathbb{R}; \mathbb{R}^n)$ is a given locally integrable function. Consider, for each $k \in \mathbb{Z}$, the Volterra type integral multioperator $\mathcal{G} : C([kT - \sigma, (k+1)T]; \mathbb{R}^n) \multimap L^1((kT, (k+1)T); \mathbb{R}^n)$ defined as

$$\mathcal{G}(u)(t) = m(t) + \int_{kT}^{t} K(t, s) F(s, u_s) ds,$$

$$\mathcal{G}(u) = \{y \in L^1((kT, (k+1)T); \mathbb{R}^n) : y(t) = m(t) + \int_{kT}^{t} K(t, s) f(s) ds : f \in \mathcal{P}_F(u)\}.$$

It is also obvious that the multioperator $\mathcal{G}$ is causal.

E x a m p l e 3. Suppose that a multimap $F : \mathbb{R} \times \mathcal{C} \to K(\mathbb{R}^n)$ satisfies the following condition of almost lower semicontinuity:

(F_L) there exists a sequence of disjoint closed sets $\{J_n\}$, $J_n \subseteq \mathbb{R}$ $n = 1, 2, \ldots$ such that: (i) $meas(\mathbb{R} \setminus \bigcup_n J_n) = 0$; (ii) the restriction of F on each set $J_n \times \mathcal{C}$ is l.s.c.

Then (see, e.g., [9, 21]) under conditions (F_L), $(F3)$, for each $k \in \mathbb{Z}$, the superposition multioperator $\mathcal{P}_F : C([kT - \sigma, (k+1)T]; \mathbb{R}^n) \multimap L^1((kT, (k+1)T); \mathbb{R}^n)$ is also well-defined and causal.

Now, suppose that $\psi \in \mathcal{C}$ is a given initial function. By the symbol D_ψ we will denote the set of all continuous functions $x : [-\sigma, +\infty) \to \mathbb{R}^n$ such that $x(t) = \psi(t),\ t \in [-\sigma, 0]$ and the restriction of x to $\mathbb{R}_+ = [0, +\infty)$ is absolutely continuous.

Considering the following abstract Cauchy problem for a functional inclusion with causal operator Q of the following form:

$$x' \in Q(x), \tag{1}$$

$$x(t) = \psi(t), \qquad t \in [-\sigma, 0], \tag{2}$$

where $x(\cdot)$ is an absolutely continuous function, we will study the problem of existence of solutions satisfying the estimate of the type

$$\|x(t)\| \le \frac{k}{g(t)}, \quad t \in \mathbb{R}_+, \tag{3}$$

where $k > 0$ and g is a given function.

2. Main results

2.1. Convex-valued causal multioperators

By the symbols L^1 and C we will denote the corresponding spaces of integrable functions $f : \mathbb{R} \to \mathbb{R}^n$ and continuous functions $x : \mathbb{R} \to \mathbb{R}^n$ with the norm $\|x\|_C = \sup\limits_{t \in [0,T]} \|x(t)\|$.

In this section we will assume that the causal multioperator $\mathcal{Q} : C \to Cv(L^1)$ has convex values and satisfies the following conditions:

($\mathcal{Q}1$) for each bounded linear operator $A : L^1 \to E$, where E is a Banach space, the composition $A \circ \mathcal{Q} : C \to Cv(E)$ is closed;

($\mathcal{Q}2$) there exists a locally integrable non-negative function $\alpha(\cdot) \in L^1_{loc}(\mathbb{R}_+)$ such that for every $x \in C$

$$\|\mathcal{Q}(x)(t)\| \le \alpha(t)(1 + \|x(t)\|) \text{ for a.e. } t \in \mathbb{R}_+.$$

To provide condition ($\mathcal{Q}1$) in Examples 1 and 2, it is sufficient to assume that the multimap F satisfies conditions $(F1)$ – $(F3)$ (see, e.g., [9, Theorem 1.5.30]) and to fulfil condition ($\mathcal{Q}2$), we can suppose, in Example 1, the following sublinear growth condition: for each $x \in C$ we have, for some non-negative integrable function $\beta(t)$:

$$\|F(t, x_t)\| \le \beta(t)(1 + \|x(t)\|) \text{ for a.e. } t \in \mathbb{R}_+, \tag{4}$$

and, in Example 2, the global boundedness condition $\|F(t, c)\| \le \gamma(t)$ for some non-negative integrable function $\gamma(t)$.

Denote by $\mathfrak{V}$ the collection of all C^1-functions $V : \mathbb{R}^n \to \mathbb{R}$ satisfying the coercivity condition $\lim_{\|x\| \to +\infty} V(x) = -\infty$. Notice that, given a function $V \in \mathfrak{V}$, for each $r > 0$ there exists $k(r) > r$ such that if $\alpha_r := \inf\{V(x),\ \|x\| \le r\}$ then $V(x) < \alpha_r,\ \|x\| \ge k(r)$.

Now, let $g : \mathbb{R}_+ \to \mathbb{R}_+$ be a given C^1-function such that $\inf\{g(t),\ t \in \mathbb{R}\} \ge 1$.

D e f i n i t i o n 5. A function $V \in \mathfrak{V}$ is called an integral guiding potential for inclusion (1) along the function g if there exists $r_V > g(0)\|\psi(0)\|$ such that for every function $x \in \mathcal{D}_\psi$ satisfying conditions

(i) there exists a largest finite number $\tau_1^x > 0$ such that $g(t)\|x(t)\| \leq r_V,\ t \in [0, \tau_1^x)$;

(ii) there exists a (least) finite number $\tau_*^x > \tau_1^x$ such that $g(\tau_*^x)\|x(\tau_*^x)\| = k_V := k(r_V)$;

(iii) $\|x'(t)\| \leq \|Q(x)(t)\|$ for a.e. $t \in \mathbb{R}_+$;

we have

$$\int_{\tau_\sharp^x}^{\tau_*^x} \big\langle \nabla V(g(s)x(s)), g'(s)x(s) + g(s)f(s) \big\rangle ds \geq 0$$

for each summable selections $f \in Q(x)$, where $\tau_\sharp^x := \sup\{\tau \in [\tau_1^x, \tau_*^x),\ \|g(\tau)x(\tau)\| = r_V\}$.

Now we are in position to formulate the main result of this paper.

Theorem 1. *If $V \in \mathfrak{V}$ is an integral guiding potential for inclusion* (1) *along the function g then each solution of Cauchy problem* (1), (2) *satisfies the estimate*

$$\|x(t)\| \leq k_V \cdot \frac{1}{g(t)}, \quad t \in \mathbb{R}_+. \tag{5}$$

2.2. Lower semicontinuous causal multioperators

In this section we will consider the Cauchy problem for a class of functional inclusions with non-convex-valued lower semicontinuous causal multioperators. Namely, we will suppose that the causal multioperator $\mathcal{Q} : C \to P(L^1)$ satisfies condition

$(\mathcal{Q}_L)$ $\mathcal{Q}$ is l.s.c. and has closed decomposable values

and condition $(\mathcal{Q}2)$.

As an example of a causal multioperator satisfying conditions $(\mathcal{Q}_L)$ and $(\mathcal{Q}2)$ we may consider the superposition multioperator $\mathcal{P}_F$ generated by a multimap $F : \mathbb{R} \times \mathcal{C} \to K(\mathbb{R}^n)$ satisfying conditions of almost lower semicontinuity (F_L) and the sublinear growth condition (4) (see, e.g., [9, 21]).

The following result holds true.

Theorem 2. *Let $\mathcal{Q} : C \to P(L^1)$ be a causal multioperator satisfying conditions $(\mathcal{Q}_L)$ and $(\mathcal{Q}2)$. If $V : \mathbb{R}^n \to \mathbb{R}$ is an integral guiding potential for inclusion* (1) *along the function g then each solution of Cauchy problem* (1), (2) *satisfies the estimate* (5).

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АСИМПТОТИЧЕСКОЕ ПОВЕДЕНИЕ РЕШЕНИЙ ВКЛЮЧЕНИЙ С КАУЗАЛЬНЫМИ МУЛЬТИОПЕРАТОРАМИ И МЕТОД ИНТЕГРАЛЬНЫХ НАПРАВЛЯЮЩИХ ПОТЕНЦИАЛОВ

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Аннотация. В настоящей работе метод интегральных направляющих потенциалов применяется для изучения задачи об асимптотическом поведении решений дифференциального включения с каузальным мультиоператором. Сначала рассматривается случай, когда мультиоператор имеет замкнутые и выпуклые значения. Затем рассматривается случай невыпуклозначной полунепрерывной снизу правой части.

Ключевые слова: функциональное включение; каузальный мультиоператор; асимптотическое поведение решений; интегральный направляющий потенциал

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